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Professor A. S. EDDINGTON, F.R.S., PRESIDENT, in the Chair.

Edward Neville da Costa Andrade, D.Sc., Ph.D., F.Inst.P., Professor of Physics, Artillery College, 18 Keyes Road, Cricklewood, London, N.W. 2;

Weld Arnold, formerly Fellow in Astronomy, Harvard University, Dunedin, Earlswood Road, Earlswood, Surrey;

Pedro Carrasco, D.Sc., Observatorio Astronómico, Madrid;

Miss Caroline Carter, B.A., Brooklyn, Lion Road, Bexley Heath, Kent;

George Stanley Le Beau, M.A., Mathematical Department, East London College, University of London, and 27 Rusthall Avenue, Bedford Park, London, W. 4;

Alexander John Thompson, B.Sc., Assistant Statistical Officer, General Register, Somerset House, London, W.C. 2;

Antonio Vela, D.Sc., Observatorio Astronómico, Madrid; and

Mrs. Lilian R. Widgery, B.A., 102 Elliscombe Road, Charlton, London, S.E. 7;

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Reginald Wheeler Dobson, M.C., 2 Gerald Road, Oxton, Birkenhead (proposed by L. J. Comrie);

Peter Doig, 96 St. George's Square, London, S.W. 1 (proposed by L. J. Comrie);

Vaman Mukund Gaitonde, M.A., Professor of Mathematics, Guzerat College of Science, Ahmedabad, India (proposed by K. B. Madhava);

Franklin Garside, 21 Cliveden Road, Hough Green, Chester (proposed by L. J. Comrie);

Thomas Herbert Le Sbirel Hony, Lloyds' Bank House, Fowey, Cornwall (proposed by H. P. Hollis);

Col. E. M. Jack, C.M.G., D.S.O., R.E., War Office, Whitehall, London, S.W. 1 (proposed by E. H. Grove-Hills);

W. M. Lindley, M.A., 26 Park Road, Rugby (proposed by F. J. M. Stratton);

Dr. Knut Lundmark, Upsala, Sweden (proposed by W. H. Wright);

C. E. Kenneth Mees, D.Sc., Director of the Research Laboratory of the Eastman Kodak Co., Rochester, N.Y., U.S.A. (proposed by L. Silberstein);

William Robert Ottway, The Niche, Penton Hook, Laleham, Staines (proposed by E. H. Grove-Hills); and

Countess Maria Vicenta de Rivas, Vernon Court, Buckingham Gate, London, S. W. 1 (proposed by A. C. D. Crommelin).

Ninety presents were announced as having been received since the last ordinary meeting, including, amongst others:—

Bordeaux Observatory: Annales, tome 16.

T. Cooke & Sons: The Adjustment and Testing of Telescopic Objectives (3rd edition).

R. Jarry-Desloges: Observations des Surfaces Planétaires, 1913 and 1914.

C. Moorsom: Meridian-Arcs: Elements and Formulæ.

Perth Observatory, Western Australia: Astrographic Catalogue, vols. 5–8.

Diffusion and Viscosity in Giant Stars.

By S. Chapman, M.A., D.Sc., F.R.S.

1. The gas in the interior of a giant star is now generally regarded as consisting of free electrons and of atoms which have lost some or all of the electrons surrounding the nucleus. The forces between the gas-particles are therefore mainly electrostatic, and vary according to the inverse square law of distance. It has been remarked by Gruner (*Ann. d. Phys.*, 35, 381, 1911) that when the law of force between the gas-particles is of this form, certain integrals occurring in the kinetic theory of diffusion and viscosity become infinite, and consideration of this point is necessary in order to estimate the coefficients of diffusion and viscosity in a giant star.

The kinetic theory of diffusion, etc., in a gas ordinarily takes account only of binary encounters between the gas-particles. This really

implies that the distance p between the lines of relative motion of two gas-particles (while still far enough apart to be moving practically in straight lines) does not exceed the mean distance d between adjacent particles of the gas. For greater values of p the probability is that one or more other particles will also be in range during the encounter of any two particles specially considered. The analytical discussion of such ternary and multiple encounters is too difficult to be attempted here: it may be inferred from general reasoning, however, and is confirmed by the agreement of theory with experiment in the case of ordinary gases, that the effect of such encounters is rightly neglected in the theory. In the present case, owing to the less rapid diminution of the force with increasing distance, multiple encounters may be of greater relative importance, but their effect will necessitate only a factorial correction to the results derived by considering binary encounters alone: for when p is taken large enough to include many particles within range of a particular one, the average effect of the others, which will be distributed nearly at random, is likely to be small unless one of them is specially close to the particular one. By confining attention to binary encounters, and taking d as the upper limit of p, results of the right order of magnitude should be obtained.

2. The integrals in question are two, and are defined as follows :—

$$I_1 = 4\pi \int \cos^2\theta \,.\, a\, da, \qquad I_2 = 4\pi \int \sin^2\theta \,.\, \cos^2\theta \,.\, a\, da,$$

where θ is half the angle through which the asymptotes of the relative paths of the particles are deflected, when the distance between the initial asymptotes is p, and their initial relative velocity is V: θ is a function of p, V, of the masses m_1, m_2, of the two particles, and of the constant k in the expression k/r^n for the force between them at distance r. In fact, writing

$$a = p\left(\frac{m_1 m_2}{m_1 + m_2} \cdot \frac{V^2}{k}\right)^{\frac{1}{n-1}}$$

we have

$$\theta = \int_0^{\eta_0} \frac{d\eta}{\sqrt{\{1 - \eta^2 - 2(\eta/a)^{n-1}\}}},$$

where η_0 is the positive root of the quantity under the radical sign.

In the present case $n = 2$, and

$$\cos^2\theta = \frac{1}{1 + a^2},$$

whether the force is attractive or repulsive. Hence

$$I_1 = 4\pi \int \frac{a\, da}{1 + a^2} = \left[2\pi \log_e (1 + a^2)\right]$$

$$I_2 = 4\pi \int \left(1 - \frac{1}{1 + a^2}\right) \frac{a\, da}{1 + a^2} = \left[2\pi \left\{\log_e (1 + a^2) + \frac{1}{1 + a^2}\right\}\right].$$

The limits of a in these integrals have not yet been specified: they are usually taken as 0 and ∞, the fact being that for the ordinary laws of intermolecular force, for which the index n exceeds 5, the part of the integrals beyond $p=d$ is negligible. But in the present case, where $n=2$, I_1 and I_2 increase indefinitely with a, and the upper limit of a must be more precisely considered.

As already explained, d will be taken as the upper limit of p, and it will be sufficient to take an average value of the factor containing V^2, *i.e.*

$$\frac{m_1 m_2}{m_1+m_2} V^2 = 3RT,$$

where T is the absolute temperature of the gas and R the universal gas-constant $1{\cdot}35 \cdot 10^{-16}$ c.g.s. If the charges on the particles are $n_1 e$, $n_2 e$, e being the electronic charge $4{\cdot}8 \cdot 10^{-10}$ e.s.u., then $k = n_1 n_2 e^2$. Hence the upper limit of a is

$$\frac{3RTd}{n_1 n_2 e^2},$$

when $n=2$. If the numbers of particles per unit volume are ν_1, ν_2 respectively, it is sufficient to take $d=(\nu_1+\nu_2)^{-\frac{1}{3}}$. For a gas of given composition, d is proportional to the inverse cube root of the density ρ, given by $(\nu_1 m_1 + \nu_2 m_2)$: the latter is proportional to T^{-3} in a giant star, so that Td, and consequently the upper limit of a, and the values of the integrals I_1, I_2, are independent of the distance from the centre of the star.

3. If there are nuclei of one kind only, and if these are completely ionised, we have $n_2=1$, the suffix 2 referring to the lighter particles, in this case free electrons. Also, if the gas is electrically neutral, $n_1\nu_1=\nu_2$, while $\rho=\nu_1 m_1$, the electronic mass being negligible. As a numerical illustration, suppose that m_1 is fifty times the mass ($1{\cdot}7 \cdot 10^{-24}$ gr.) of the hydrogen atom, $n_1=25$, and that $\rho=0{\cdot}1$, $T=7 \cdot 10^6$, as at the centre of the typical giant star considered in my earlier paper on this subject (*M.N.R.A.S.*, **77**, 540, 1917). On substitution, the upper limit of a proves to be approximately 16, while $I_1=33$ approximately.

The coefficient of diffusion (D) between the nuclei and electrons is, to a first approximation, given by

$$\frac{3}{\nu_1+\nu_2}\left(\frac{\pi}{2}\cdot\frac{m_1+m_2}{m_1 m_2}\right)^{\frac{1}{2}} \frac{(RT)^{\frac{5}{2}}}{I_1 n_1^2 n_2^2 e^4},$$

when the law of force is that of the inverse square. With the above numerical values inserted, this proves approximately equal to 100.

The coefficient of viscosity κ of the mixture can be calculated from equation (17.02) of my paper on the kinetic theory of mixed gases (*Phil. Trans.*, A, **217**, 189, 1917), putting $\nu_2=n_1\nu_1$, and using equations (4.15, 9.27, 9.30) and those of § 9 (*f*). Since m_1/m_2 is very great in the present instance, many terms are negligible in the expression

(17.02), which reduces to $\rho D/2n_1$ approximately. The kinematic viscosity κ/ρ is therefore $D/2n_1$, which in the above numerical illustration, where $D = 100$ roughly, is about 2. This is about a hundred times the kinematic viscosity of water. Such a degree of viscosity would have very little influence in slowing down large-scale convective motions in a star (*cf.* Lamb's *Hydrodynamics*, 3rd ed., p. 580).

The above expressions show that, as T diminishes with increasing distance from the centre of a star, D and κ/ρ will increase proportionately to $T^{-\frac{1}{2}}$, so long as the composition of the gas remains the same.

These estimates assume that the ordinary kinetic theory is approximately applicable to a gas under the conditions found within a giant star. In spite of the high pressure and somewhat high density, this assumption is probably admissible, in view of the correspondingly high temperature. On account of the different properties assigned to the gas, the above estimates of D and κ/ρ differ from those regarded as probable in my former paper (*l.c.*) on convection and diffusion in giant stars.

4. A quantity which is of more interest than those considered in § 3 is the rate of diffusion, relative to one another, of charged atoms of different weights, in the midst of the electrons which render the gas neutral as a whole. This quantity is required in the discussion of a problem suggested to the writer by Professor H. N. Russell, viz. as to the length of time taken for atoms to diffuse from one part of a star to another region distant by an assigned fraction of the radius of the star. If in the interior of a star there is some substance which at high temperatures undergoes some atomic change, resulting in the liberation of a large amount of energy and so supplying the loss by radiation, the rate of supply will depend on the speed with which, by diffusion or otherwise, the exhausted substance is replenished from the cooler parts of the star. Since the transformed substance seems most likely to be hydrogen,* which would naturally tend to rise to the outer regions of the star, diffusion can supply the deficiency at the centre only if the relative concentration of this substance diminishes towards the centre even more rapidly than it would do in a state of diffusive equilibrium, such as was considered in my former paper (*l.c.*: the results of that paper do not depend on the actual magnitude of the coefficient of diffusion).

An exact treatment of the problem, involving the consideration of three sets of particles (*i.e.* electrons and the two sets of nuclei), is too difficult to be attempted, when, as in this case, we may be content with an estimate of the order of magnitude of D, without knowing its actual value. The order of magnitude is given by $\lambda\bar{c}$, where λ is the mean free path of a particle and $\bar{c}$ is the mean velocity. The usual formula † for λ would indicate that in this case the mean free path of a nucleus is determined almost entirely by encounters with electrons, and not with other nuclei, of either set: this is because of the small mass and high velocity of the electrons. Their momentum is small, however, and at collision they do not much affect the momentum of

* *Cf.* Eddington, *Observatory*, **43**, 354, 1920.

† *Cf.* Jeans' *Dynamical Theory of Gases*, 3rd ed., p. 252.

the nuclei. This fact can be allowed for by taking into account the "persistence of velocities," and the expression for λ, so corrected, is of the same order of magnitude as that obtained by considering collisions between nuclei alone (*cf.* Jeans' treatise, p. 275). Hence it seems sufficient to calculate the order of magnitude of D by ignoring the presence of the electrons: since the latter must certainly hinder the diffusion of the nuclei, the result will be an overestimate of the rate of nuclear diffusion.

The formula for D is as before, except that m_1 no longer denotes the mass of a hydrogen atom, to which the suffix 2 now applies: also $\rho = \nu_1 m_1 + \nu_2 m_2$, the second term no longer being negligible. If the atomic weight of the heavier nuclei is N, we have $\rho = (N\nu_1 + \nu_2)m_1$. Knowing N, m_2 and ν_1/ν_2, we could calculate $\nu_1 + \nu_2$ from ρ. Suppose for numerical illustration that $N = 4$, $\nu_1 = 2\nu_2$, $\rho = 0{\cdot}1$: then, since $m_2 = 1{\cdot}7 \cdot 10^{-24}$, we have $\nu_1 + \nu_2 = 2 \cdot 10^{22}$: also $d = (\nu_1 + \nu_2)^{-\frac{1}{3}} = 4 \cdot 10^{-8}$ approximately. If $n_1 = 2$, $n_2 = 1$, the value of the upper limit of a (T being $7 \cdot 10^6$ as before) is roughly 250, and the corresponding value of I_1 is 70. The estimate of D resulting from these quantities is, in round numbers, 300. If the heavier nuclei were of greater atomic weight, the value of n_1 being correspondingly greater, D would be lessened (mainly owing to the factor n_1^2 in the denominator of D, although $\nu_1 + \nu_2$ is diminished), since $D \propto T^{-\frac{1}{2}}$, its value at half-radius in the star, where the temperature is about one-third that at the centre, is nearly doubled.

It may be noted that the decrease in D as n_1 increases may be regarded as owing to an increase in the equivalent diameter of the nuclei, as the field of force becomes more intense and consequently, for a given value of the repulsive force, more extensive.

5. If the giant star is at first composed entirely of hydrogen, the transformation of this into heavier elements initially involves no difficulty in the maintenance of the supply of hydrogen: the heavier element remains at the centre, but until it has accumulated so as to leave only a small fraction of hydrogen in the central region, the hydrogen is replenished simply by the natural process of settling down over the heavy element. The problem of diffusion as a means of supplying losses due to the transformation becomes of interest only when the heavy gas is predominant in the region of transformation. Diffusion will occur when the rate of diminution of the proportion of light gas, towards the centre, is in excess of the natural rate of diminution. The latter rate was discussed, with numerical illustrations, in the former paper cited in § 3. If the *excess* rate is $-\partial'\lambda_2/\partial r$, where λ_2 denotes the proportion, by volume, of the light gas, the velocity of diffusion will be $D\partial'\lambda_2/\partial r$, and the rate of flow of mass of the light gas is $D\rho_2\partial'\lambda_2/\partial r$. As the light gas gets used up near the centre, this rate diminishes owing to the factor ρ_2, representing the density of the lighter component. The problem is too indefinite to allow of more than numerical illustration, and for this purpose we will suppose that within a certain radius r', λ_2 diminishes uniformly to zero at the centre: the velocity of diffusion will be overestimated by treating it as $D\partial\lambda_2/\partial r$, *i.e.* by ignoring the part of the gradient of λ_2 which would be present

in a state of diffusive equilibrium. The rate of flow of lighter gas, by mass, across a sphere of radius r ($<r'$) is therefore

$$4\pi r^2 . D\rho_2(\partial\lambda_2/\partial r) = 4\pi r^2 . D\rho_2\lambda_2'/r',$$

λ_2' being the value of λ_2 at r'. The density ρ_2 is less than $\lambda_2\rho$ or $\lambda_2' r\rho/r'$ in a ratio depending on the atomic weight of the heavier nuclei: let this ratio be k. Then the last expression becomes

$$4\pi r^3 k . D\rho(\lambda_2'/r')^2,$$

which can supply matter to this sphere at the rate per unit volume $3kD\rho(\lambda_2'/r')^2$ increasing towards the centre owing to the factor ρ.

If this rate of flow of lighter gas across the sphere r is measured as a fraction of the total mass of the star, it becomes

$$3kD(\rho/\bar{\rho})(r/r_0)^3(\lambda_2'/r)^2,$$

where r_0 denotes the whole radius, and $\bar{\rho}$ the mean density. Suppose, for example, that $r = r' = r_0/3 = \frac{1}{3} . 7 . 10^{11}$ cm., $\bar{\rho} = 0{\cdot}002$, $\rho = 0{\cdot}02$, $D = 300$, $\lambda_2' = \frac{1}{2}$, and that the heavier gas has atomic weight 4: then $k = \frac{1}{5}$, and the ratio just mentioned becomes roughly $3 . 10^{-22}$, reckoned in seconds as the unit of time, or 10^{-15} if the year is the unit. Thus it would take 10^{13} years for a quantity of the light gas, equal in mass to 1 per cent. of the whole mass of the star, to diffuse across the said sphere. This calculation is both very rough and very special, but it seems likely that it illustrates the order of magnitude of the power of diffusion to supply light gas to the central region of the star, at the stage when $\lambda_2 = \frac{1}{2}$ at one-third radius: at this stage, with the given mass-ratio of the nuclei, the lighter gas would probably make up about a quarter of the whole mass of the star: in the earlier stages, the rate of transfer by diffusion would be greater than the above, but it should be noticed that if the mass-ratio of the two sets of nuclei were increased, the above estimate would be diminished, since both D and ρ_2 would be reduced.

Variation of Latitude at Greenwich in 1921.

(*Communicated by the Astronomer Royal.*)

The preliminary results for variation of latitude at Greenwich during 1921, given in the accompanying table, have been derived from the measurement of plates obtained with the Cookson floating zenith telescope.

The plates were grouped in periods roughly corresponding to tenths of the year, and the number of plates associated with the respective quantities found for the variation is denoted by subscript.

The abrupt change of latitude suspected to have occurred during the summer, reported by Herr M. Schnauder in *Ast. Nach.*, No. 5134, to amount to over $0''{\cdot}5$, and partially supported in a later communication

from Signor J. Boccardi (*Ast. Nach.*, No. 5138), is not confirmed by the Greenwich results.

Fraction of Year.	Mean − Observed Latitude.
	″
1921·04	+0·166$_{8}$
·17	+ ·041$_{23}$
·27	+ ·089$_{12}$
·37	+ ·006$_{22}$
·44	− ·113$_{21}$
·50	− ·106$_{22}$
·57	− ·093$_{17}$
·67	− ·113$_{24}$
·78	− ·034$_{36}$
·93	+ ·120$_{19}$
1922·03	+ ·211$_{21}$
·16	+ ·094$_{24}$

The following values for the variation have been read from a smooth curve drawn through the observed values plotted against the times of observation, due regard having been paid to the weight of each point :—

Fraction of Year.	Mean − Observed Latitude.
	″
1921·0	+0·086
·1	·088
·2	·066
·3	+ ·022
·4	− ·040
·5	·117
·6	·118
·7	− ·079
·8	+ ·002
·9	·090
1922·0	·140

Observations of Nova Cygni (No. 3).

By P. M. Ryvês.

The short series of estimates of the brightness of Denning's *Nova* made by the writer had been reserved for detailed publication together with those of other observers; the following numerical data (sufficient for plotting) are now given, however, since further delay seemed undesirable.

The star was observed on 29 nights between 1920 August 21–September 22 (inclusive). Of the 47 Argelander observations obtained, the first 21 were made without optical aid, the last 11 with Galilean binoculars magnifying three diameters, the remainder by both means except where indicated. Magnitudes of comparison stars have been taken from *Revised Harvard Photometry* (*H.A.*, **50**), with some slight alterations. Grade value has been adjusted and corrections applied for atmospheric absorption and for position-angle effect where this had not been eliminated. The weights (1–5) are based on conditions of observation, number of comparisons, and probable error, and in the absence of fuller details due attention should be given to them. The worst observations should be discarded when others made about the same time are available. The mean divergence for a single comparison is ±0·064, and the mean σf the P.E.s of those (24) observations which consist of more than three comparisons is ±0·024.

J.D. G.M.T. 2,422,5	H.R.	W.	R.	J.D. G.M.T. 2,422,5	H.R.	W.	R.	J.D. G.M.T. 2,422,5	H.R.	W.	R.
58·38	3·23	1	*i*	67·62	4·02	2	*ill*	78·35	5·23	5	*o*
59·35	2·76	4	*o*	,, ·65	4·00	3	*ll*	,, ·58	5·22	3	
,, ·47	2·84	4	*o*	69·35	4·07	2	*i*	79·35	5·26	2	*o*
,, ·67	2·64	1	*it*	,, ·46	4·11	2	*l*	80·36	5·48	2	*ob*
61·35	1·84	3	*l*	71·55	4·3±	1	*l*	,, ·46	5·55	2	*on*
,, ·39	1·88	4		72·33	4·46	2		,, ·47	5·44	1	*ob*
,, ·58	1·95	3	*f*	,, ·36	4·48	3	*o*	81·64	5·57	1	*f*
62·48	2·42	3	*fl*	,, ·56	4·50	2	*l*	82·47	5·56	1	
63·35	2·78	4	*l*	73·44	4·66	2	*n*	84·31	5·67	1	
,, ·44	2·79	5	*l*	74·43	4·80	4	*o*	,, ·55	5·77	1	*o*
,, ·65	3·02	3		75·35	4·75	2	*i*	85·40	5·8	1	
64·45	3·21	4		76·45	4·88	3	*on*	86·31	5·97	1	
65·36	3·40	3	*fll*	,, ·45	4·90	2	*ob*	87·31	6·12	1	
,, ·43	3·54	3	*fll*	77·34	5·14	3		88·57	6·0	½	*ii*
66·38	3·71	1	*ill*	,, ·57	5·13	4	*o*	89·49	6·30	2	
,, ·44	3·77	1	*ill*					90·36	6·47	2	*l*

Remarks.—*l* = moonlight, *t* = twilight, *f* = fog or haze, *i* = cloud, *o* = transparent dark sky, *b* = binocular only, *n* = naked eye only.

There are slight indications of an oscillation in brightness near the time of maximum with a period of about 1 day. Later, from 7 to 22 days after max., more definite fluctuations can be traced. Secondary

maxima are indicated near J.D. 2422576·0. ...79·4 and ...83·3; secondary minima about ...67·6, ...74·1, ...77·7, and ...81·1. A period of $3\frac{1}{2}$ days would satisfy the observations about this time, but at a season of short observing nights a period differing little from one day "heterodyned," so to speak, by the earth's rotation period, might produce a somewhat similar result with the observations obtained at a single station. On this supposition a period of $1^d·4$ would suit the observed magnitudes.

Though the departures from a smooth curve indicated by the above observations do not exceed one-tenth of a magnitude, they undoubtedly represent real fluctuations in the light of the *Nova*, the precise nature of which can only be determined from an examination of numerous light-estimates made in different longitudes.

A New Variable Star in Cygnus.
By A. Stanley Williams. (Plate 7.)

B.D. + 34° 4217 (9·4 mag.).

R.A. = 20h 52m 24s Decl. = +34° 37′·1 (1855).
R.A. = 20h 54m 12s Decl. = +34° 47′·4 (1900).

This star does not appear hitherto to have been recognised as variable. Its variability was first noticed by the writer some time in the year 1905, when comparing two photographs taken with a 4·4-inch portrait lens. On one plate, dated 1900 October 27, $12^h\ 15^m$ to $12^h\ 50^m$, G.M.T., the star was nearly half a magnitude fainter than on a number of other plates, so that it seemed likely that it was a variable of the Algol-type. Visual observations with a $6\frac{1}{2}$-inch reflector were commenced in October last, and these soon showed that the period was short (about $15^h\ 10^m$), and that besides the principal minimum there was a secondary minimum, the light-curve resembling that of β Lyræ.

Table I. gives the names and places (for 1855) of the comparison stars, with the light-scale and the adopted magnitudes of the stars.

TABLE I.

Star.	B.D. No.	R.A.	Decl.	B.D. Mag.	Light-scale.	Adopted Mag.
	°	h m s	° ′			
H	+34 4222	20 53 02	+34 56·7	8·8	38	8·85
D	+34 4216	20 52 08	+34 47·9	9·1	29	9·30
A	+34 4215	20 51 32	+34 52·9	9·0	23	9·60
b	...	20 52 11	+34 36	...	13	10·10
c	...	20 52 06	+34 34	...	12	10·15
d	...	20 52 37	+34 41	...	0	10·75

The variable was only directly compared with the last three stars. The places of these are very rough, but will serve for identification.

The observed heliocentric Greenwich mean times of principal minimum are given in Table II. A weight 3 in column 4 signifies that the star was well observed both during decreasing and increasing light; 2 during either the decrease or increase: and 1 that it was not well observed in either phase. The first minimum of the table is photographic, being the middle of the times of exposure of the photograph previously alluded to. The second minimum is derived from a single observation only. The times were obtained by means of a mean light-curve nearly identical with the one described later on.

TABLE II.

E.	Observed Min.	Computed.	Wt.	O − C.
				d
(−12115)	2415320·525	·6811	ph.	(−0·1561)
0	2422967·4339	·4268	1	+ ·0071
16	2977·5298	·5257	2	+ ·0041
30	2986·3603	·3622	1	− ·0019
38	2991·4062	·4116	3	− ·0054
41	2993·3068	·3052	3	+ ·0016
52	3000·2430	·2482	2	− ·0052
60	3005·2957	·2976	2	− ·0019
117	3041·2750	·2749	2	+ ·0001
125	3046·3311	·3243	1	+ ·0068
136	3053·2694	·2673	3	+ ·0021
155	3065·2590	·2597	3	− ·0007

The adopted elements of variation are:

Heliocentric G.M.T. of principal minimum = J.D. 2422967·4268 + $0^d·63118$ E.

The residuals O − C, according to these elements, are stated in the last column of Table II. The average residual, without regard to sign, is $^d·0034$ or $4^m·9$. The principal minimum is so easy to observe that this is large, but the weather was bad. In fine weather the average error of a single observation ought not to exceed $\pm 2^m$. The photograph of 1900 October 27 is no help at present for the purpose of correcting the period, for the computed time of principal minimum might apply equally well both to that minimum or to the secondary minimum.

Between 1921 October 4 and 1922 January 10, 219 observations were made of the brightness of the variable on 28 nights. For deriving the mean light-curve, 33 of these were rejected, because made with sky only = 1. They were probably all more or less affected by cloud. Also 6 observations made near the secondary minimum on October 17 were rejected, because it was thought that they might be affected by systematic error. They made the minimum unduly faint. The remaining 180 observations were formed into 36 groups (5 observations to a group)

according to phase, counting from the principal minimum. The means of these groups are given in Table III.

TABLE III.

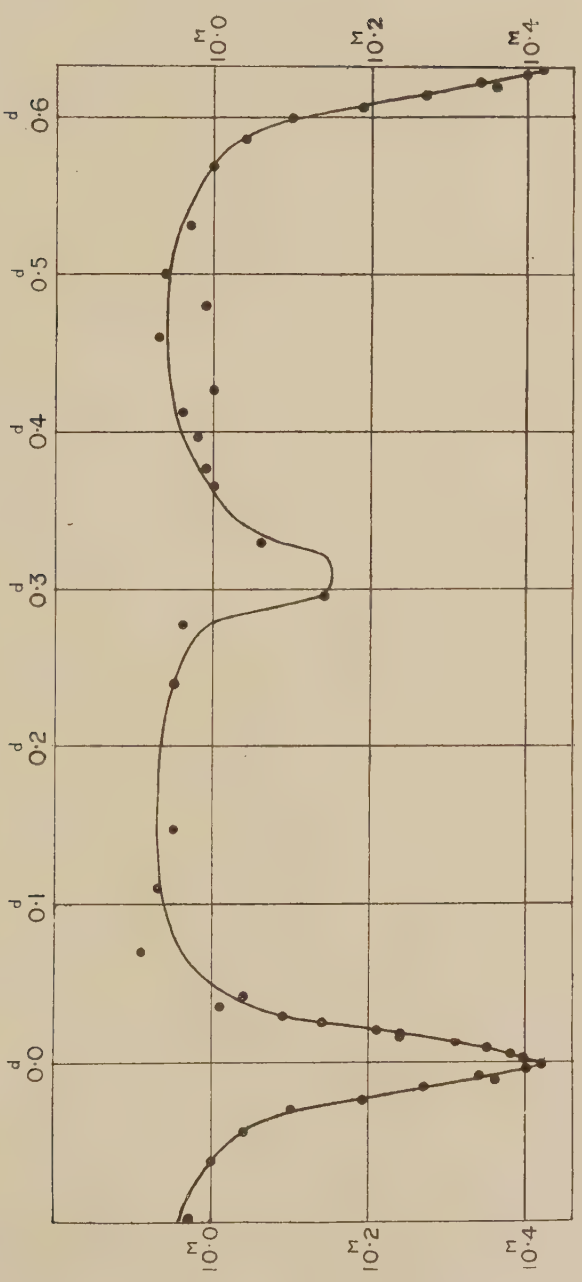
Phase.	Mag.	Phase.	Mag.	Phase.	Mag.	Phase.	Mag.
d		d		d		d	
0·0022	10·40	0·0350	10·01	0·3646	10·00	0·5678	10·00
·0050	10·38	·0422	10·04	·3774	9·99	·5860	10·04
·0092	10·35	·0692	9·91	·3966	9·98	·5998	10·10
·0130	10·31	·1100	9·93	·4122	9·96	·6060	10·19
·0156	10·24	·1472	9·95	·4252	10·00	·6142	10·27
·0182	10·24	·2404	9·95	·4596	9·93	·6192	10·36
·0204	10·21	·2766	9·96	·4796	9·99	·6222	10·34
·0246	10·14	·2954	10·14	·4990	9·94	·6264	10·40
·0292	10·09	·3294	10·06	·5306	9·97	·6300	10·42

The diagram (Plate 7) shows the above results with the mean light-curve derived therefrom. It will be seen that the general form of the curve is clearly that of the β Lyræ type, though the eclipse is apparently not quite total in the case of the present star. But though the principal minimum is well observed, the observations are not sufficiently numerous elsewhere to show satisfactorily the exact forms of the secondary minimum and the two maxima. The discordances are not, however, abnormally large, considering the bad weather, the magnitude scale being an extended one. The most obtrusively discordant result is that at ^{d}·0692, but even here the disagreement with the mean curve drawn barely exceeds half a tenth of a magnitude.

Summary.

Period	$15^{h}\ 08^{m}\ 53^{s}\cdot95$
Principal minimum . . .	10·42 mag.
First maximum	9·93 ,,
Secondary minimum . . .	10·15 ,,
Second maximum	9·94 ,,
Range	0·49 ,,

Polvarth, St. Mawes:
1922 *Feb.* 15.



Light Curve of Variable in Cygnus.

Occultations of Stars observed during the Lunar Eclipse of 1921 *October* 16. By G. Merton and W. H. Steavenson.

The following occultations were observed by us at Woldingham during the large partial eclipse of 1921 October 16. The instrument used was the 6-inch Cooke refractor, with a power of 200. The seeing was good, and the disappearances were in every case abrupt. Times were taken from the beats of the sidereal clock, whose error was determined by wireless time signals from Paris both before, during, and after the eclipse.

The longitude of the observatory is $0^h\ 0^m\ 8^s{\cdot}74$ W., and the latitude $51^\circ\ 17'\ 51''{\cdot}1$ N.

Star.	Mag.	Phenomenon.	G.M.T.	Observer.
			h m s	
263_B Piscium	6·4	Disapp.	8 54 46·4	W.H.S.
B.D. +7°·214	8·5	,,	8 57 34·4	W.H.S.
B.D. +7°·218	9·2	,,	10 2 43·9	G.M.
		Reapp.	10 59 2·0	G.M.
B.D. +7°·224	9·0	Disapp.	10 18 0·2	W.H.S.
B.D. +7°·222	9·3	,,	10 19 10·5	W.H.S.
B.D. +7°·225	9·5	,,	10 26 51·2	W.H.S.
B.D. +7°·228	9·3	,,	10 56 34·4	W.H.S.
B.D. +7°·227	9·1	,,	11 5 11·5	W.H.S.

New Zealand Observations during the Lunar Eclipse 1921 *April* 21.

(*Communicated by L. J. Comrie.*)

Guided by predictions in *B.A.A. Journal*, 31, 147, the following observations of occultations were made during the above total lunar eclipse. The times are in G.M.T.

B.D. −10°	Phase.	Hector Observatory.	Wellington College.	Stratford.	New Plymouth.	Hawera.
		h m s	h m s	h m s	h m s	h m s
3812	R	19 30 48·4	19 31 12		19 33 55	19 25 32
3813	R	19 50 32·2	19 50 37		19 52 11·5	19 48 43
3817	D	19 20 37·6	19 20 33	19 17 15		Cloudy
3818	D		19 41 10	19 36 28		
3819	D	19 22 37·8	19 22 57	19 19 36		Cloudy
3819	R	20 25 48·8	20 25 31			Cloudy
3820	D	19 40 42·5	19 40 52	19 36 56		19 37 50
3823	D	19 55 44·6	19 55 47	19 54 25		19 53 0

Notes on stations, observers, etc.:—

Hector Observatory, Wellington: 174° 46′ 4″ E., 41° 17′ 3″·8 S., 420 feet above sea-level. Observer, C. E. Adams; Recorder, Capt. G. Hooper.

Wellington College Observatory: 174° 47′ 7″ E., 41° 18′ 10″ S., 200 feet above sea-level. Observer, A. C. Gifford.

Stratford Observatory: 174° 17′ 37″ E., 39° 21′ 28″ S., 995 feet above sea-level. Observer, Mr. Burrell.

New Plymouth Observatory: 174° 4′ 27″ E., 39° 3′ 45″ S. Observer, Rev. O. Blundell.

Hawera Observatory: 174° 18′ E., 39° 36′ S. Observer, G. M. Townsend.

Dr. C. E. Adams, Government Astronomer, says: "I used the 5½-inch Grubb telescope . . . a number of other members were also busy round the Observatory. It was, therefore, a matter of considerable difficulty to give close attention to the observations. The same remark applies to Mr. Gifford's observations, so I would not like these to be published as 'Precise Observations.'"

Stars occulted during Lunar Eclipses in 1921.

By L. J. Comrie, M.A.

The occultations observed during the total lunar eclipse of 1921 April 21, and published elsewhere in this number, together with those observed during the partial eclipse of 1921 October 16, and published on pp. 57 and 303 of this volume, will be of service in supplying a correction to the Moon's tabular position when the places of the stars are fairly accurately known. These stars are, with two or three exceptions, deficient in reliable meridian observations, so I put forward the suggestion that they should be placed on the programme of several observatories. A table of the approximate positions for 1900·0, with Newcomb's precessions, is appended. The magnitudes marked * are from the Henry Draper Catalogue; the others are from the A.G. Catalogue, if the star appears there, or from the B.D.

Notes on the Stars.

\+ 7°213 = 263 B Piscium = Lalande 2677 = Pi I^{h} 85.
\+ 7 214 = Pi I^{h} 87. The P.M. is towards the north-east.
\+ 7 218. P.M. towards the south-east.
\+ 7 224. The B.D. declination is 1′·4 greater.
\+ 7 225. The B.D. declination is 0′·5 greater.
\+ 7 227. Comparison of A.G. and B.D. indicates a considerable P.M.
− 10 3818. May be too faint for meridian observation.

Star B.D.	Mag.	R.A. 1900·0.	Ann. Prec.	Sec. Var.	Dec. 1900·0.	Ann. Prec.	Sec. Var.
+7°			+	+	+	+	−
		h m s	s	s	° ′ ″	″	″
213	6·4*	1 23 8·3	3·1343	0·0099	7 26 36	18·742	0·170
214	8·7*	1 23 12·8	3·1343	0·0099	7 26 25	18·740	0·170
218	8·7	1 24 16·6	3·1387	0·0102	7 51 56	18·707	0·172
222	9·3	1 25 7·0	3·1389	0·0102	7 49·0	18·680	0·174
224	9·4*	1 25 17·9	3·1374	0·0101	7 37 44	18·674	0·174
225	9·5	1 25 27·3	3·1388	0·0102	7 46 15	18·669	0·175
227	8·9	1 26 5·3	3·1411	0·0103	7 59 10	18·649	0·176
228	9·0	1 26 13·8	3·1393	0·0102	7 46 11	18·645	0·176
−10°			+	+	−	−	+
3812	8·5*	13 58 33·7	3·1983	0·0128	10 47 32	17·424	0·238
3813	9·3*	13 59 2·3	3·2015	0·0129	11 0 53	17·403	0·239
3817	10·0*	14 0 7·5	3·2024	0·0129	11 0·0	17·356	0·241
3818	9·9	14 0 12·4	3·2050	0·0130	11 12·9	17·352	0·241
3819	9·6*	14 0 12·2	3·2025	0·0129	11 0 29	17·352	0·241
3820	9·6*	14 0 41·3	3·2043	0·0130	11 6 44	17·331	0·242
3823	7·9*	14 1 21·4	3·2039	0·0130	11 1 26	17·301	0·243

A Detonating Fireball in Sunshine, 1922 *February* 7, $3^h\ 55^m$.
By W. F. Denning.

On February 7, $3^h\ 55^m$, when sitting in a position facing north, and the sky being of that beautiful blue colour which it sometimes displays even in our unfavourable climate, I observed a brilliant meteor falling in due north at a considerable angle (roughly 45°) towards the N.N.W. point of the horizon. The object fluctuated both in size and luminosity, and its motion appeared irregular, as though impeded by encountering air strata of different densities. In fact, the flight suggested a series of spurts, each one reviving the brilliancy and evolving a stream of bright yellow-red sparks. The nucleus was of a pale golden tint gleaming attractively and with almost startling vividness in its setting of deep azure.

When it had reached an altitude of 21° and an azimuth of 6° W. of N. the meteor, after crumbling into pieces, became finally reduced to a mere spark. I carefully noted positions, and, in the evening, when the stars appeared, obtained the path (allowing for time interval) as

Beginning 197° + 69½°, ending 210° + 60°.

This probably represents the concluding section of the flight, for when the object first caught my eye it seemed in full combustion and

must have been near mid-career. About 4½ minutes after its extinction I heard the sound of a distant explosion northward, with reverberations continued for several seconds. I was immediately led to associate the noise with the disruption of the meteor, as it was very dissimilar to the sounds of blasting, etc., usually heard in the district.

I received a number of observations from people in various parts of the country, and make a few extracts :—

Mr. Charles Hill, Fayre Oak, Hereford, writes he saw the fireball at 3.56, and it appeared to come from the direction of the moon, which had risen high in the E. The meteor travelled at great speed in a succession of darts, immature explosions were taking place, throwing sparks of beautiful colours, and the speed seemed retarded intermittently. The object, however, acquired fresh impetus, and disappeared into the haze above what seemed the N.W. horizon. It left a long trail of white smoke which was plainly visible until 4 o'clock. At 3.59 there was a tremendous explosion, more definite than thunder, in the exact direction in which the phenomenon disappeared, and the rumbling ceased at precisely 4 o'clock.

Miss Margaret E. Walters of Axbridge, 15 miles S.W. of Bristol, says that "the fireball was most brilliant, traversing, after it caught my eye, a slight arc a little to my right, facing north (N. by E.). In fact, it was very high, and appeared almost over my head. From a map of the district I found that the end of the meteor's flight must have been due north."

Mr. W. J. B. Rees of Greenmeadow, near Cardiff, describes the object as "looking like a ball of bright fire. It descended northwards a short distance, shooting out, upwards, sheets of flame as it fell. The altitude of middle part was about 47°, and inclination of descent 60°, duration 3 to 5 seconds. It was too wonderful a sight not to be recorded."

Mr. W. Woodyatt of Hereford says: "I was working in the greenhouse at Brampton Bryan Hall on February 7, just before 4 p.m., and looking up at the roof, when I saw what appeared to me as a ball of fire in the sky. As the glass was steamy, I ran outside to get a clearer view, but when I looked up again it had vanished."

A Bristolian, walking along Redcliff Backs at 3.55, observed a fireball of great brilliance travelling towards the north.

Mr. E. S. Darmady of Sherborne, Dorset, writes that on February 7, $3^h\ 57^m$, he saw a flash of fire in the sky almost due N., and about 20° above the horizon. The direction was from E.N.E. It had a comparatively short track, and the nucleus divided into two portions, in shape like the triangular points of a spear.

The detonations of the meteor were heard at the residence of Miss Agnes Fry, F.R.A.S., Failand, about 6 miles S. of Bristol. But they were much louder near those places over which the object passed. Reports have come from Edgbaston (Miss Sylvia M. Groves), from Feckington, Droitwich, and Mere Hall (Mr. H. R. Hodgkinson), about 20 miles S. of Birmingham. At Quinton (12 miles S. of Warwick) the noise was of appalling intensity. The ground vibrated, people were greatly startled, and cattle and horses galloped about the fields, according to Mr. A. W. Keep (*Birmingham Daily Post*, February 13).

I have compared the various reports both of the positions and detonations of the fireball, and find that the radiant point is fairly well indicated at 60° – 11°. The height of the object was from 56 to 32 miles, along a course of 80 miles from above a point near Oxford to above Shropshire. The velocity was about 14 miles a second. The direction of motion was from south-east to north-west, the position of the radiant point being in south-east, altitude 17°.

This radiant point does not appear to have been previously observed as a centre of meteoric divergence in the month of February. The position near γ Eridani has been found at Bristol as an active radiant of meteors both from October 21 to 28 and early in January.

In the spring of 1921 two large fireballs were observed from apparently the same positions, their radiant points being March 29, 62° – 10°, and April 10, 64° – 11°.

Perhaps the occurrence of daylight fireballs is not so great a rarity as is generally supposed. I have compiled a list of twelve good examples of these bodies, observed from the United Kingdom during the last twenty-one years.

Bristol:
February 25.

Meteoric Phenomena, February 7–22. By W. F. Denning.

A series of remarkable meteoric appearances have been observed, chiefly of late years, during the above period of 16 days.

1922 February 7, 3.55 p.m. A brilliant fireball giving a loud detonation observed in sunshine from Bristol and many other places. Radiant 60° – 11°.

1894 February 8, 0.28 p.m. A fireball in sunshine observed from many places. It moved swiftly from a radiant in Hercules. On 1863 February 7 a large fireball as bright as the moon was also seen from a radiant in Hercules (265° + 28°).

1913 February 9. Fireball observed from radiant in Perseus at 47° + 46°.

1913 February 9. A stream of brilliant fireballs observed from Canada. They were followed over about 5500 miles from Saskatchewan over the North Atlantic Ocean to Cape la Roque on the east coast of Brazil. Their height was about 42 miles, and velocity 8 miles a second. They were virtually moving as satellites of the earth during the time they were visible, and completed more than one-fifth of a revolution.

1896 February 10. A bright flash and meteoric explosion occurred over Madrid and fragments of a meteorite fell near. The streak or luminous debris of the object remained visible in the sunlit sky for 5½ hours.

1905 February 11. Three large fireballs observed in England from 10.22 to 11.45 p.m. Radiants 145° – 13°, 150° – 7°, and 166° + 33°. On 1850 February 11 a fireball larger than the moon was observed from a radiant at 37° + 34°.

1871 February 13. Fireball brighter than the moon observed from Bristol and other places. Radiant in Hydra.
1901 February 13. Fireball seen from radiant near Capella.
1906 February 13. Fireball seen from a radiant in the Lynx ($105° + 51°$).
1910 February 17. Large fireball from radiant near Capella.
1920 February 17. Fireball from near Capella.
1920 February 17. Large fireball from radiant $261° + 60°$.
1922 February 17. Large fireball observed from radiant $125° + 13°$ in Cancer.
1911 February 19. Fireball with a long flight of 590 miles observed from radiant at $46° - 15°$.
1898 February 20. Large fireball from radiant $176° + 12°$ near β Leonis.
1894 February 21. Fireball observed from radiant at $141° + 52°$. Seen at Bristol and other places.
1909 February 22. Large fireball seen and reported by over 200 observers from radiant $196° + 20°$. It passed over English Channel and left a bright streak which underwent remarkable changes but remained visible over two hours. It was about 53 miles high, and drifted to north-west on an upper wind current at a velocity of 120 miles per hour. 1879 February 22: a large fireball was directed from a radiant at $140° + 5°$ and was only $5\frac{1}{2}$ miles high at the end point.

It is true that records of fireballs have now accumulated to such an extent, and may be found in the B.A. "Luminous Meteor" Reports (1848–1881) and in other publications, that it is possible to find a number of fireballs for any epoch of the year, but the *very exceptional objects* seen in February appear to deserve special notice. The list might be greatly extended if it were carried back some years.

I have sometimes endeavoured to discover the special showers visible at certain apparently suggestive periods from the real paths of fireballs observed, but except in a few prominent instances the attempt has not been successful. Thus from 40 fireballs seen in February I found that 34 different radiants must have been implicated. There were but two showers, represented by 3 or 4 paths each, namely, $73 + 42$ and $147 - 11$ (February 7–17 and February 11–13). In fact, the investigation proved that the meteor flights had a remarkable diversity of motion, and attested the very large number of feeble systems in activity. Observations of ordinary small meteors and shooting stars are quite in accord with this deduction.

Bristol:
February 25.

Observations of the Meteors of the ϵ Arietid Radiant in 1921.

By Miss A. Grace Cook and J. P. M. Prentice.

The following table gives the determinations of the position of the ϵ Arietid radiant obtained during the course of extensive meteor watching carried on through the autumn of 1921 by (D) Mr. W. F. Denning, (C) Miss A. Grace Cook, and (P) Mr. J. P. M. Prentice, and other observers. It is thought that this set of determinations affords striking confirmation of the stationary character of some meteor radiants, which was announced by Mr. Denning many years ago.

Our success in observing this long duration of the radiant must be attributed to the special attention which has been given to the shower during this period at the expense even of richer showers.

The earliest record of this shower is on 1921, July 10 (noted here): the latest is that of a meteor doubly observed on 1909, January 25. The small gaps in this table are well filled by observations obtained by others in previous years.

It should be noted that every effort has been made to obtain a good number of paths each night and not to combine the results over a long period.

TABLE.

Date.	Radiant.	Number of Meteors.	Observers
July 10	41°+20°	6	P.
10	42+20	* F.P.	C. & P.
Aug. 1–2	40+20	5	P.
Sept. 7–8	43+20	10	P.
8	43+22	3	C.
25, 27	42+23	7	P.
Oct. 5	42+23	4	P.
6	43+23	4	C.
8, 9	43+22	3	C.
16	45+22	† S.S.P.	C. & E. W. Foster.
25	43+21	† S.S.P.	C. & E. H. Collinson.
23, 25	43+21½	8	C.
24, 27	43+21	8	P.
27, 29	44+21	10	D.
Nov. 4, 6	43+19½	14	D.
4, 7	43+21	4	C.
6, 8	43+21	9	P.
22–25	43+21	15	C.
22–26	43+21	15	P.
23	40+22	† S.S.P.	P. & S. B. Mattey.
Dec. 18	41+24	* F.P.	6 observers.
20, 22	43+22	6	P.
20, 23	43+22½	4	C.
28	43+22	4	P.
31	43+21	2	C.

* Fireball path. † Shooting star path.

Star Discs. (3), *The Effect of Coma.* By L. C. Martin, D.I.C., A.R.C.S., B.Sc., Lecturer in the Optical Engineering Department, Imperial College of Science, Technology, South Kensington.

(*Communicated by Professor A. E. Conrady.*)

Review of Theory.—In a previous paper on Star Discs,* Professor Conrady gives certain results of his investigations regarding the actual distribution of light in a star image when "coma" is present in the optical system. The intensity of light is shown to be unsymmetrical with regard to the focussing point of the central zone of the convergent wave, and therefore the effects of this aberration, if present, are extremely important in regard to the measurement of stellar photographs and the deduction of the relative angular positions of stars.

A scheme of work recently undertaken at the Imperial College has aimed at the comparison of theoretical results with actual effects shown by lens systems exhibiting definite amounts of aberration. The first work in regard to spherical aberration has already been published and the beginning of the study of coma is described in the present paper.

A quick theoretical review will elucidate the methods employed. The general expression for the *relative* acceleration W of a wave showing aberrations with regard to the ideal wave having a uniform curvature equal to that of the central zone is given by Professor Conrady : †

$$W = a_1(x^2+y^2)^2 + a_2x(x^2+y^2)V + a_3(3x^2+y^2)V^2 + a_5xV^3 + a_6V^4.$$

We adopt the notation of that paper and assume a_1, a_3, a_5, and a_6 zero in order to study the coma term only. Then,

$$W = a_2x(x^2+y^2)V,$$

and it is at once seen that the wave has a constant curvature along the line $x=0$.

Using the polar co-ordinates for the location of the reference point in the wave surface the expression becomes $W = a_2S^3V\cos E$. Referring to fig. 1 we notice that a small displacement σ parallel to the x axis to the point p from the extra axial image point F_1 introduces a further relative phase difference W′ between disturbances arriving from a zone of the wave surface of radius S and from the centre given (when V is not large) by

$$W' = \frac{-\sigma S\cos E}{P}, \text{ (neglecting } \sigma^2),$$

where P is OF_1 the radius of curvature at the centre of the wave.

Then if we take $\sigma = Pa_2S^2V$ there will be uniformity of phase amongst all disturbances arriving at the point p from a zone of radius S, or, in

* *Monthly Notices, R.A.S.*, June 1919.
† *Ibid.*, Jan. 1920, p. 324.

other words, the wave surface is, for that zone, at a constant distance from p.

An interesting model to represent the divergence of the wave from the spherical form is made by hingeing a number of annular rings about a common diameter and tilting successive rings through small angles proportional to the squares of the radii.

The relative positions of the series of "points of uniform phase" for successive zones (with regard to the geometrical ray intersection points in the focal plane) is obtained from the ray directions resulting from the

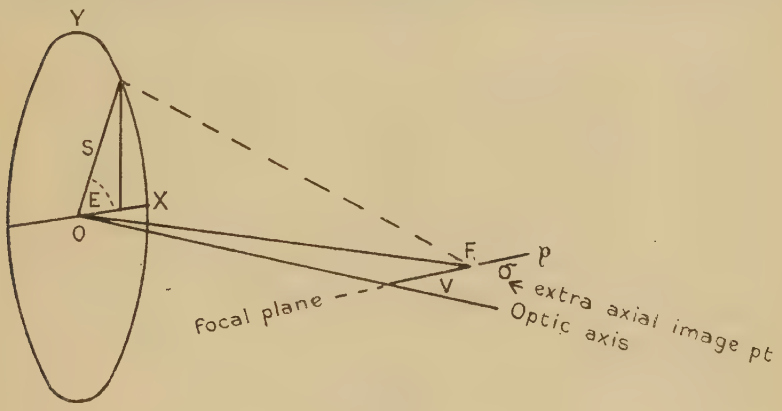


FIG. 1.

differentiation of the equation to the wave surface. As far as the coma term is concerned (see page 326 of Professor Conrady's paper), we have

"X" displacement of a ray in the image plane $= Pa_2 S^2V(2 + \cos 2E)$.
"Y" " " " " $= Pa_2 S^2V \sin 2E$.

Evidently the above "uniform phase" points (found, of course, on the x axis) correspond to the points passed through by a ray when $\cos 2E = -1$ or when $E = \pi/2$.

Method of Integration.—The method of mechanical integration resulting from such conclusions with regard to coma is described in the first paper on star discs. In order to obtain quick results the figures of Table II. of that paper were plotted on a large scale to facilitate interpolation.

To anticipate a result described later in this paper it was found that the wave surface produced by an experimental lens gave a coma value such that the extreme W values of the marginal zone correspond approximately to ± one wave-length, or $\pm 360°$. It was desired to obtain theoretical results which could be checked practically.

Since the results in the original paper were obtained by using 16 zones enclosing equal areas it was decided to integrate for a path difference of $\pm 352°$ or $22° \times 16$. Focussing points of successive zones will then

be found by 16 successive equal steps away from the head of the figure, *i.e.* between H and T_S in fig. 2. By the term "focussing point" we indicate the point of uniform phase for a particular zone.

The extreme phase difference ($\pm$ W) between the disturbances arriving at this point from *another* zone will, however, vary in proportion to the separation of the "focussing points" and also with the relative radii of the zones.

Thus in obtaining the amplitude contributions of successive zones to add as in Table II. it is necessary to take, not only a vertical step corresponding to the change of radius, but also a horizontal step corresponding

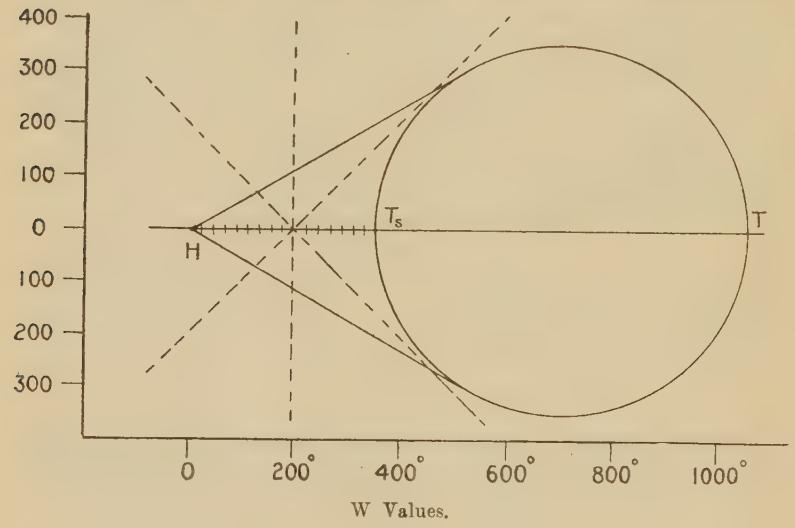


FIG. 2.

to the change of "focussing point." Thus on going inside the figure to a point corresponding to, say, the focussing point of the third zone from the centre (66°) we have

r^2 =	0	1/16	2/16	3/16	4/16	5/16	etc.
corresponding $\overline{W}$ =	66°	44°	22°	0°	22°	44°	etc.

The successive contributions for the zones are thus read off from the curves and added to obtain (as they are already multiplied by the required numerical factors) the final values of amplitude.

For a point away from this line the W values of the successive zones are proportional simply to its distance from the corresponding focussing points and are found at once graphically by plotting on a scale of W values, see fig. 2.

In the Table I. the points along the line are defined by the zonal W value with reference to the focussing point of the central zone.

TABLE I.

W. in Degrees.	Amplitude. (Sum.)	Intensity. (1/10 sum squared.)
−132°	1·451	·021
− 88	1·941	·038
− 44	2·273	·051
0	2·836	·080
44	3·611	·131
88	4·914	·241
132	5·977	·356
176	7·014	·491
220	7·812	·610
264	7·748	·598
330	5·106	·261
396	1·502	·023
462	−1·714	·029
506	−2·799	·078
528	−3·024	·091
550	−2·844	·081
616	−1·244	·015

(The values in this table are shown in fig. 3.)

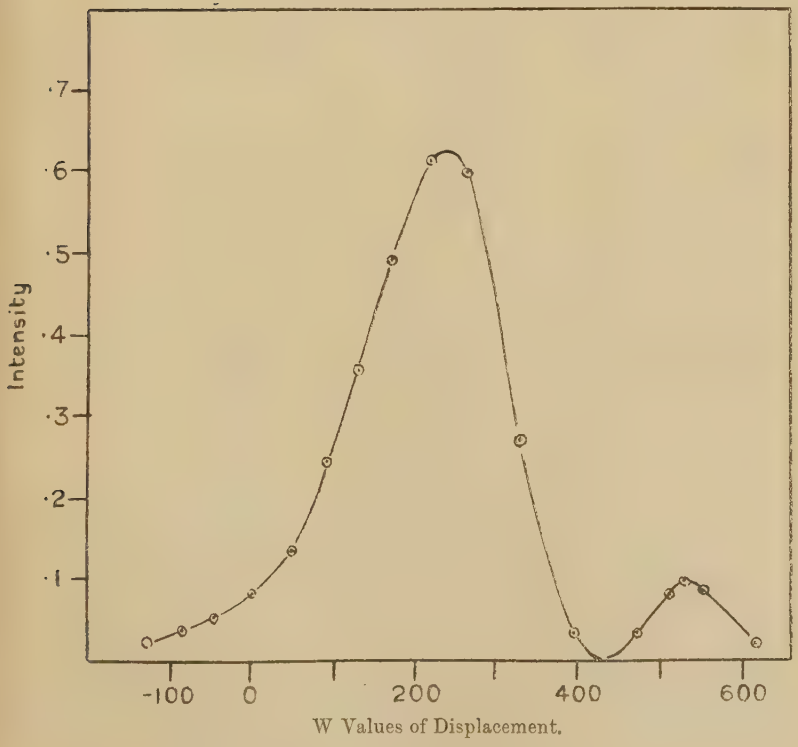


FIG. 3.—Distribution of Intensity on Axis of Coma Patch.

The result is interesting, as it indicates that the maximum intensity of the illumination in the image remains as high as ·6 of the value when no aberration is present, thus showing that the extreme residuals of phase aberration are less effective in destroying the concentration of the light than in the case of spherical aberration. (This we should expect from the uniformity of phase in the neighbourhood of the $x=0$ diameter, which is not modified by coma.)

From rough integration it is found that the "centre of gravity of the curve" lies at $W=210$ in the present case. As in the other cases investigated by Professor Conrady, the falling off in intensity is rather more gradual towards the head of the coma figure ($W=0$) and there appears to be a dark ring and a bright ring beyond it towards the tail. We should expect to find second dark and bright rings if the integration were carried far enough. The first bright ring is, however, about five times as bright as the first ring in the Airy disc when aberration is absent.

The extra axial points selected were on two lines passing through the point $W=198°$ at 90° and 45° respectively to the original line of symmetry. The results obtained are given in Tables II. and III.

TABLE II.

(Section at right angles * through $W=198°$.)

Displacement.	Amplitude.	Intensity.
W= 0	7·50	·560
50	6·58	·42
146	2·342	·055
240	−0·962	·0092
320	−1·151	·013
400	− ·023	·00005

TABLE III.

(Line through $W=198°$ at 45° to first line.)

Displacement Values.	Amplitude.	Intensity.
−263°	0·867	·008
−174	2·575	·066
−105	4·626	·213
0	7·55	·560
+ 38	7·367	·539
+107	5·463	·297
+213	− ·407	·002
+285	−2·772	·076
+320	−2·927	·085
+390	−1·590	·025

* The section is, of course, symmetrical on each side of the first line. See fig. 4.

The intensity of the bright ring towards the tail of the figure (fig. 5) is shown to be well maintained between the two lines at 45° to the axis for which the above figures are valid. The quickest falling off in the ring occurs between the lines at 45° and 90° to the axis.

From the above results it is possible to obtain a fairly complete idea of the distribution of light, and, in order to exhibit the theoretical results in the most suggestive manner, lines* are drawn through the

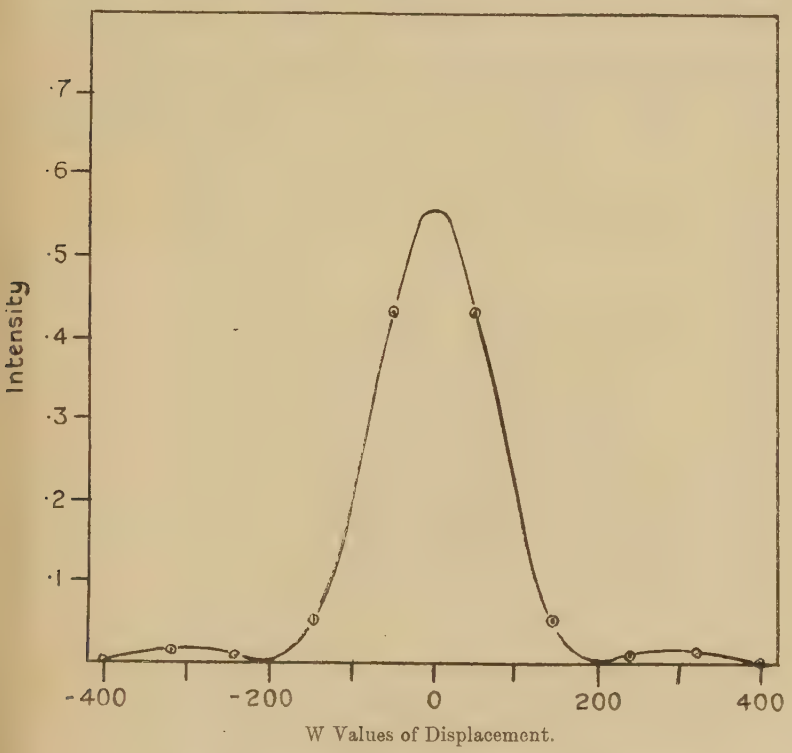


Fig. 4.—Intensity on Perpendicular through W = 198°.

points of equal brightness, thus making a kind of contour map exhibiting luminous intensities. This is shown in fig. 6 as given by the seven or more such points in any contour line obtained from the figures.

The contours for ·6, ·5, ·4, ·3, ·2, ·1, 0·5, 0·25, and 0 are shown. The contour line of zero intensity (dotted in the figure) is approximately hyperbolic.

At right angles to the axis of symmetry the falling off in intensity from the "highest" point is very similar to that in the "Airy disc," but on the axis the slope is greater than this value towards the bright

* Such lines have been termed "isolumes."

ring (the tail of the coma) and less towards the focussing point of the central zone. The dimensions of the "Airy disc," on the same scale, are indicated, and, in order to exhibit the comparison between the results of physical integration and geometrical ray tracing, part of the geometrical coma figure is indicated in its relative correct position and size in the contour map.

The "Centre" of the Image.—The point of greatest practical concern in the present investigation is the location of the effective "centre" of the image. It is evident that such a "centre" may vary with the

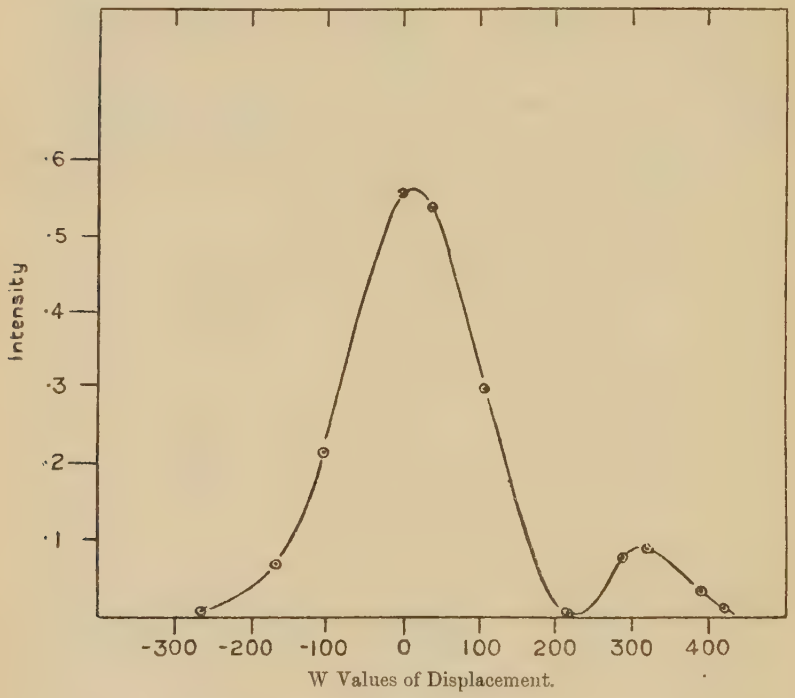


FIG. 5.—Intensity on Line at 45° to Axis of Coma Patch through W = 198°.

practical means employed for recording it, and there is no absolute criterion available. The "centre of gravity" of the light distribution is, however, sometimes used for theoretical purposes: it will be of interest to consider the difference in the location of this point by physical and geometrical methods. To begin with, the centre of gravity found by physical methods undoubtedly lies near the maximum ordinate on the central line of symmetry at W = 250°, or a little nearer the "head" of the figure.

In the geometrical figure the ray distribution for a particular zone gives a circular track, the centre of gravity of which must be at the centre, since the distribution of rays round the circle takes

place at a uniform rate with continuous variation of E. To show this we have

$$\frac{d\,(\text{X displacement})}{d\text{E}} = -2\text{P}a_2\text{S}^2\text{V}\sin 2\text{E}$$

and

$$\frac{d\,(\text{Y displacement})}{d\text{E}} = 2\text{P}a_2\text{S}^2\text{V}\cos 2\text{E},$$

and therefore

$$(d\text{X})^2 + (d\text{Y})^2 = \text{const}\,(d\text{E})^2.$$

Since the distribution of the centres of the circles of uniform "weight" is at equal linear intervals, the centre of gravity of the entire figure must lie at T_8 (fig. 2), or $\text{W} = 352°$.

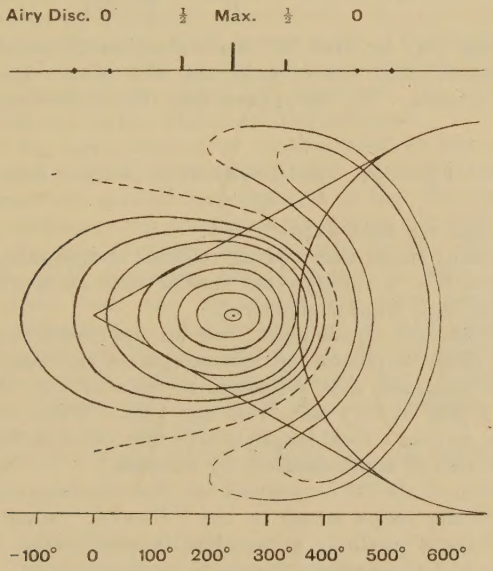


FIG. 6.—Light Distribution "Contours" in the Presence of Coma.

The discrepancy between the two theoretical positions is therefore comparable with half the radius of the Airy disc.

Practical Tests.—As a practical check on the theoretical work I have computed a doublet microscope objective (the main design of which is based on a system obtained by Professor Conrady in the course of the development of certain analytical methods) which is free from objectionable spherical aberration and astigmatism while leaving a considerable residual of coma. The system has been made up and yields results confirming the above theory when tested on the "artificial stars" found in a silver film deposited on a microscope slide. It is hoped shortly to publish a description of the practical results obtained.

Coma and the "Sine Condition."—The offence against the sine

condition corresponding to the amount of coma in the above lens was about double that which is usually considered negligible in modern computing practice, although many telescope objectives of older types contain larger amounts of aberration than this. The focussing points for different zones found on the basis of the sine condition are (as Professor Conrady points out) identical with those "points of uniform phase" to which reference is made above.

Writing M_m and M_p for the marginal and paraxial magnifications for a zone of radius S we have therefore

$$\sigma = V\left(\frac{M_m}{M_p} - 1\right) = Pa_2S^2V.$$

$$\therefore \quad W = a_2S^3V\cos\theta = \frac{S\sigma\cos\theta}{P} = \frac{SV}{P}\left(\frac{M_m}{M_p} - 1\right)\cos\theta.$$

This formula may be used for the evaluation of the phase residuals when the aperture ratio, size of field, and the offence against the sine condition are known. In the above case the following values were obtained:

$$\left(\frac{M_m}{M_p} - 1\right) = \cdot 004. \quad \text{(At full aperture, diam.} = 1\cdot 09 \text{ cms.)}$$

Angle of principal ray with axis $= 2^\circ\ 45'$.

It is then easily found that a phase residual of approximately $\pm 360^\circ$, corresponding to $W = \pm \cdot 000059$ cms. results when an aperture of $\cdot 9$ cms. (considering the back lens) is employed.

A type of doublet object glass with an approximately equiconvex crown lens is likely to possess an offence against the sine condition of one part in a thousand with an aperture ratio of $\frac{1}{15}$. The W value then rises to $\pm 360^\circ$ at $1\cdot 77$ cms. from the axis. With a $12''$ objective of such a type, having a focal length of 457 cms., this corresponds to an angle with the axis of approximately 13 minutes.

In conclusion, it should be pointed out that the foregoing theoretical results apply to the image found in the "Petzval" image surface only. Any change of focus produces a considerable modification in the distribution of light, and these effects remain to be investigated.

The following Obituary Notice was received too late for inclusion in the Annual Report:—

Thomas Lawrence Bennett was born on 1882 October 25 at Sudbury in Middlesex. He was educated at Christ's Hospital and Trinity College, Cambridge. He graduated in mathematics, being 7th Wrangler in 1904. In the following year he took Part II. of the Mathematical Tripos, and was placed in Class I., division 2.

In 1905 July Bennett entered the service of the Egyptian Government in the computation office of the Survey Department under Capt. (now Col.) H. G. Lyons. In addition to survey work, the computation office was then charged with the collection and discussion of meteorological as well as hydrological observations. In 1912 he was put in charge of the computation of all survey work. During the war, owing to the absence of the greater portion of his colleagues, greatly increased responsibilities were thrown upon his shoulders. In 1920 January he was made Controller of the Statistical Department.

Owing to the large amount of routine work he was called upon to do, Bennett had little time for the pursuit of original research. He, however, carried out several investigations on the theory of errors. In the realm of astronomy his energies were mainly directed towards those branches of the science which are required by the surveyor and the geodesist. He was, however, always ready to help his colleagues engaged in other work, and his advice was often sought; when any problem was submitted to him, he attacked it with vigour and would seldom lay it aside until he had settled it to his own satisfaction.

His untimely death, which took place on 1921 February 9, robbed the Egyptian Government of one of its most brilliant servants. He leaves a widow and two children.

He was elected a Fellow of the Society on 1910 January 14.